

# VARIATIONAL MASTER EQUATION FOR A DRIVEN QUANTUM DOT EMBEDDED IN AN OPTICAL RESONATOR

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(Dated: November 18, 2016)

In this work, we study the dynamics of a quantum dot embedded in a QED cavity and driven by a coherent electromagnetic field, while interacting with a bath of acoustic phonons. We address the phonon dissipative effects on the resonance fluorescence of the system by deriving a variational master equation, valid for a broader range of temperatures, pumping rates, and quantum dot-cavity coupling intensities, than previously reported approaches.

## INTRODUCTION

Semiconductor quantum dots (QDs) embedded in microcavities have established themselves as a new paradigm in cavity quantum electrodynamics (cavity-QED). Continued progress in the design and fabrication of these semiconductor nanostructures leads to practical applications in quantum information processing, including the efficient generation of indistinguishable photons.

A growing number of experimental studies have focused on the resonant fluorescence of InGaAs quantum dots coupled to high quality microcolumn type cavities; Which have provided a clear demonstration of induced excitation [1, 2]. Thus, in systems with presence of a non-resonant coupling between the laser and the cavity, the cavity mode is excited efficiently by the emission of photons from a quantum point coupled to the environment of acoustic phonons [3, 4]. The inverse effect of non-resonant coupling has also been observed, where the quantum point is excited through the emission of photons from the cavity [5].

Dara PS McCutcheon and Ahsan Nazir developed a variational master equation to describe the dynamics of a two-level system in contact with a cavityless boson environment, which was applied in the study of Rabi's rotations of a quantum dot [6]. In particular, they found that the technique of variational master equation captures effects generally considered to be non-perturbative, such as multiphoton processes and renormalization of the Rabi frequency induced by the phonon bath; obtaining reliable results in regimes in which the weak and polaronic models fail.

## THEORY

The dynamics of a quantum dot embedded in a cavity is modeled considering an exciton as a quantized electron-hole system, where the electron occupies a state in the conduction band and the hole occupies a state of the valence band. Neglecting spin, the dominant feature of a

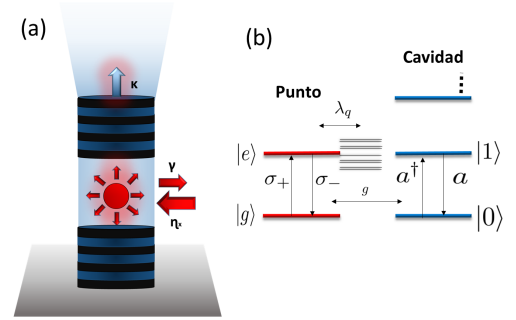


FIG. 1. (a) Schematic of a quantum dot coupled to a micropillar cavity and forced laterally by a CW laser (pumping rate  $\eta_x$ ). (b) Energy diagram of an exciton embedded in a cavity, where  $|e\rangle$  denotes the excited state and  $|g\rangle$  the ground state of the QD, while  $|1\rangle$  y  $|0\rangle$  represent the first excited and ground state of the cavity mode.

strongly confined quantum dot is described by the two lower energy states. This two-level model is conditioned by the interaction of the electron with the environment vibration modes, i.e. acoustic phonons. The dot-cavity system is driving by a continuous wave laser (CW), as shown in the figure 1(a) while the diagram of energy levels associated with it is outlined in the figure 1(b).

Working in a rotating frame with respect to laser frequency  $\omega_L$ , the Hamiltonian which describes a QD-cavity system that interacts with a bath of acoustic phonons is written

$$\begin{aligned}
 H = & \hbar\Delta_{XL}\hat{\sigma}^+\hat{\sigma}^- + \hbar\Delta_{CL}\hat{a}^\dagger\hat{a} \\
 & + \hbar\eta_x(\hat{\sigma}^+ + \hat{\sigma}^-) + \hbar g(\hat{\sigma}^+\hat{a} + \hat{a}^\dagger\hat{\sigma}^-) \\
 & + \hat{\sigma}^+\hat{\sigma}^- \sum_q \hbar\lambda_q(\hat{b}_q + \hat{b}_q^\dagger) + \sum_q \hbar\omega_q\hat{b}_q^\dagger\hat{b}_q.
 \end{aligned} \tag{1}$$

The phonon bath is represented by a continuum of frequency harmonic oscillators  $\omega_q$  being its annihilation (creation) operator  $b_q(b_q^\dagger)$ , whose constants of coupling with the exciton are represented by  $\lambda_q$  (assumed real); also the detuning of the frequency of the exciton is introduced ( $\omega_x$ ) and the cavity ( $\omega_c$ ) to the pumping frequency of the laser  $\Delta_{\alpha L} = \omega_\alpha - \omega_L$  ( $\alpha = x, c$ ). The annihilation

(creation) operator of photons of the cavity is  $a(a^\dagger)$  while the exciton Pauli operators are  $\sigma^-, \sigma^\dagger$ , and  $g$  the exciton-cavity coupling constant. The pumping rate  $\eta_x$  describes the coherent driving of the CW laser on QD and equals half of the classical frequency of Rabbi.

Let us consider a generalization of the polaron transformation, that displaces the bath oscillators according to the state of the QD excitonic system, by an amount that is determined by a set  $\{f_q\}$  of variational parameters. The variational transformation can be written as

$$H_v = \exp(S) H \exp(-S), \quad (2)$$

where

$$S = \hat{\sigma}^+ \hat{\sigma}^- \sum_q \frac{\nu_q}{\omega_q} (\hat{b}_q^\dagger - \hat{b}_q). \quad (3)$$

The transformed Hamiltonian becomes

$$H_S = \hbar \Delta_{XL} \hat{\sigma}^+ \hat{\sigma}^- + \hbar \Delta_{CL} \hat{a}^\dagger \hat{a} + \hbar \hat{\zeta}_x, \quad (4)$$

$$H_I = \hat{\sigma}^+ \hat{\sigma}^- \sum_q \hbar \lambda_q (\hat{b}_q + \hat{b}_q^\dagger) = \hbar \hat{\zeta}_{zz} B_z, \quad (5)$$

$$H_B = \sum_q \hbar \omega_q \hat{b}_q^\dagger \hat{b}_q, \quad (6)$$

with

$$B_\pm = e^{\pm \sum_q \frac{\nu_q}{\omega_q} (\hat{b}_q^\dagger - \hat{b}_q)} \quad (7)$$

$$B_x = \frac{1}{2} (B_+ + B_- - 2\langle B \rangle), \quad (8)$$

$$B_y = \frac{1}{2i} (B_+ - B_-) \quad (9)$$

$$B_z = \sum_q (\lambda_q - f_q) (\hat{b}_q^\dagger + \hat{b}_q) \quad (10)$$

$$(11)$$

The variational shift,

$$R = \sum_q \omega_q^{-1} f_q (f_q - 2\lambda_q) \quad (12)$$

and the thermal average of the bath displacement operator

$$\langle B_\pm \rangle = \exp \left[ -\frac{1}{2} \sum_q \frac{f_q^2}{\omega_q^2} \coth(\beta \hbar \omega_q / 2) \right]. \quad (13)$$

where  $J(\omega)$  is the phonon spectral function defined later in Equ. (20). For a dot-driven system,  $\hat{\zeta}_x$ ,  $\hat{\zeta}_y$  and  $\hat{\zeta}_a$  are defined through

$$\hat{\zeta}_x = \eta_x (\hat{\sigma}^+ + \hat{\sigma}^-) + g (\hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger) \quad (14)$$

$$\hat{\zeta}_y = \eta_x (\hat{\sigma}^+ - \hat{\sigma}^-) + g (\hat{\sigma}^+ \hat{a} - \hat{\sigma}^- \hat{a}^\dagger) \quad (15)$$

$$\hat{\zeta}_z = \hat{\sigma}^+ \hat{\sigma}^- \quad (16)$$

## FREE ENERGY MINIMIZATION

Before we go on to derive our variational master equation, we must first determine the free parameters  $\{f_q\}$ . We choose them such that they minimize the free energy associated with the variationally transformed Hamiltonian. At zero temperature this corresponds to minimizing the ground-state energy, as usual. At finite temperature, as the free energy is minimized in equilibrium, the variational transformation then attempts to find the best possible diagonalization of the complete Hamiltonian, given the restricted form of this unitary.

To minimize the free energy, we compute the Feynman-Bogoliubov upper bound given by

$$A_u = -\frac{1}{\beta} \ln \left( \text{Tr} \left\{ e^{-\beta H_0} \right\} \right) + \langle H_{I_v} \rangle_{H_0} + O(\langle H_{I_v}^2 \rangle_{H_0}), \quad (17)$$

where  $H_0 = H_{S_v} + H_{B_v}$  y  $\langle H_{I_v} \rangle_{H_0} = \text{Tr} \{ H_{I_v} e^{-\beta H_0} \}$ .  $A_u$  is the upper limit of the real free energy  $A$ , Which is related to the latter through inequality  $A \leq A_u$ . On the other hand,  $H_{I_v}$  has been constructed in such a way that the second term in equation (17) is null, in addition are neglected the orders greater than  $H_{I_v}$ . Inasmuch as  $[H_{B_v}, H_{S_v}] = 0$ ,  $A_u$  can be simplified and divided into two parts

$$A_u = A_E - \frac{1}{\beta} \ln \left( \text{Tr} \left\{ e^{-\beta H_{S_v}} \right\} \right), \quad (18)$$

with  $A_E$  the free energy of the phonon bath. Since  $A_E$  is not a function of  $f_q$ , is neglected in minimizing the process. In this way you will find Equ. 19.

where

$$\nu_1 = \sqrt{\Delta_{CL}^2 + \Delta_R^2 + 2B^2(g^2n + 2\eta_x^2) + 2\bar{U}},$$

$$\nu_2 = \sqrt{\Delta_{CL}^2 + \Delta_R^2 + 2B^2(g^2n + 2\eta_x^2) - 2\bar{U}}$$

and

$$\bar{U} = \sqrt{(B^2g^2n - \Delta_{CL}\Delta_R)^2 + 4B^2(B^2g^2n + \Delta_{CL}^2)\eta_x^2},$$

$$\text{with } \Lambda_1 = \frac{B^2g^2n(g^2n + 4\eta_x^2) - \Delta_{CL}(g^2n\delta_R - 2\Delta_{CL}\eta_x^2)}{\bar{U}}$$

$$\text{and } \Lambda_2 = \frac{\Delta_{CL}(\Delta_{CL}\delta_R - B^2g^2n)}{\bar{U}}.$$

Each variational parameter differs for each wave vector  $\mathbf{q}$ . In particular for the wave vectors  $\mathbf{q}$  Whose corresponding frequencies satisfy  $\eta_x/\omega_k \ll 1$  y  $g/\omega_k \ll 1$ , the minimization condition approximates  $f_q \rightarrow \lambda_q$ . For these modes, the oscillator bath can follow the excitonic movement becoming fully shifted when the system is in the excited state. In the opposite case for  $\eta_x/\omega_q \gg 1$  and  $g/\omega_q \gg 1$  It is found that  $f_q$  becomes very small. In this situation, the oscillations are so rapid that the relevant modes of the phonon bath can not follow them, and their displacements are suppressed.

$$f_q = \frac{\lambda_q \left[ 1 - \frac{2(\delta_R + \Lambda_2) \sinh(\frac{\beta}{2} \nu_1)}{\nu_1 (\cosh(\frac{\beta}{2} \nu_1) + \cosh(\frac{\beta}{2} \nu_2))} - \frac{2(\delta_R - \Lambda_2) \sinh(\frac{\beta}{2} \nu_2)}{\nu_2 (\cosh(\frac{\beta}{2} \nu_1) + \cosh(\frac{\beta}{2} \nu_2))} \right]}{1 - \frac{2(\delta_R + \Lambda_2) \sinh(\frac{\beta}{2} \nu_1)}{\nu_1 (\cosh(\frac{\beta}{2} \nu_1) + \cosh(\frac{\beta}{2} \nu_2))} \left( 1 - \frac{g^2 n + 2\eta_x^2 + \Lambda_1}{\omega_q (\delta_R + \Lambda_2)} \right) - \frac{2(\delta_R - \Lambda_2) \sinh(\frac{\beta}{2} \nu_2)}{\nu_2 (\cosh(\frac{\beta}{2} \nu_1) + \cosh(\frac{\beta}{2} \nu_2))} \left( 1 - \frac{g^2 n + 2\eta_x^2 + \Lambda_1}{\omega (\delta_R - \Lambda_2)} \right)}, \quad (19)$$

Writing  $f_q = \lambda_q F(\omega_q)$  and defining the spectral density  $J(\omega) = \sum_q |\lambda_q|^2 \delta(\omega - \omega_q)$ , which for acoustic phonons is defined as

$$J_{ph}(\omega) = \alpha \omega^3 e^{-\omega^2/2\omega_b^2}, \quad (20)$$

we find the following self-consistent forms for  $\langle B \rangle$  and  $R$  in the continuum limit

$$R = \int_0^\infty \frac{J(\omega)}{\omega} F(\omega) (F(\omega) - 2) \quad (21)$$

$$\langle B \rangle = \exp \left[ -\frac{1}{2} \int_0^\infty \frac{J(\omega)^2 F(\omega)^2}{\omega^2} \coth(\beta \hbar \omega / 2) \right]. \quad (22)$$

### MASTER EQUATION

We next introduce the Born-Markov master equation for the reduced density operator,  $\rho(t)$ , of the cavity-QED system in the second-order Born approximation of the system-reservoir coupling. In the interaction picture, we consider the exciton-photon-phonon coupling  $H_I$  in the Born approximation and trace over the phonon degrees of freedom. Phenomenologically, we also include the radiative decay of the QD and the cavity mode decay as Liouvillian superoperators acting on the reduced system density matrix; in addition, we incorporate a ZPL pure dephasing process beyond the IBM with a rate  $\gamma'$ , which is also known to increase linearly with temperatures. These Lindblad superoperators that act on the reduced system density matrix are defined through

$$\begin{aligned} \mathcal{L}(\rho) = & \frac{\gamma}{2} (2\hat{\sigma}^- \rho \hat{\sigma}^+ - \hat{\sigma}^+ \hat{\sigma}^- \rho - \rho \hat{\sigma}^+ \hat{\sigma}^-) \\ & + \kappa (2\hat{a} \rho \hat{a}^\dagger - \hat{a}^\dagger \hat{a} \rho - \rho \hat{a}^\dagger \hat{a}) \\ & + \frac{\gamma'}{2} (2\hat{\sigma}_{11} \rho \hat{\sigma}_{11} - \hat{\sigma}^+ \hat{\sigma}_{11} \rho - \rho \hat{\sigma}^+ \hat{\sigma}_{11}) \end{aligned} \quad (23)$$

In this way the variational master equation takes the form

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -\frac{i}{\hbar} [H_s, \rho(t)] + \mathcal{L}(\rho) \\ & - \int_0^t d\tau \sum_{\substack{l=x,y,z \\ m=x,y,z}} C_{lm} [\hat{\zeta}_m, e^{-iH_s \tau / \hbar} \hat{\zeta}_l e^{iH_s \tau / \hbar} \rho(t) \\ & + \int_0^t d\tau \sum_{\substack{l=x,y,z \\ m=x,y,z}} C_{lm}^* [\rho(t) e^{-iH_s \tau / \hbar} \hat{\zeta}_l e^{iH_s \tau / \hbar}, \hat{\zeta}_m] \end{aligned} \quad (24)$$

where  $C_{lm}(\tau) = \langle B_l(\tau) B_m \rangle$  with  $l, m = x, y, z$  and we have assumed that the phonon bath is in thermal equilibrium. The correlation functions are

$$C_{yy} = \langle B \rangle^2 (\cos \phi(\tau) - 1) \quad (25)$$

$$C_{xx} = \langle B \rangle^2 \sin \phi(\tau) \quad (26)$$

$$C_{zz} = \int_0^\infty d\omega J(\omega) [1 - F(\omega)]^2 \quad (27)$$

$$\times (\cos \omega \tau \coth(\beta \omega / 2) - i \sin \omega \tau) \quad (28)$$

$$C_{zy} = \langle B \rangle \int_0^\infty \frac{J(\omega)}{\omega} F(1 - F) \quad (29)$$

$$\times (i \cos \omega \tau + \sin \omega \tau \coth(\beta \omega / 2)) \quad (30)$$

and

$$C_{yz} = -\langle B_z(\tau) B_y(0) \rangle, \quad (31)$$

$$C_{xz} = C_{zx} = C_{xy} = C_{yx} = 0, \quad (32)$$

which depend on the phonon correlation function,

$$\phi(\tau) = \int_0^\infty \frac{J(\omega)}{\omega^2} F(\omega)^2 (\cos \omega \tau \coth(\beta \omega / 2) - i \sin \omega \tau), \quad (33)$$

### MASTER EQUATION ON THE LIMBLAND FORM

The variational master equation [Eq. (11)] includes both coherent and incoherent contributions from electron-phonon scattering, but some care and insight is needed in extracting the relevant incoherent scattering rates. It is therefore instructive to construct a simplified phonon-modified master equation of the Lindblad form, we do this by simplifying the term,  $e^{-iH_s \tau / \hbar} \hat{\zeta}_m e^{iH_s \tau / \hbar}$ ,

appearing in the full Born-Markov master equation. The resulting Lindblad form master equation enables a very simple numerical solution and facilitates the extraction of various phonon-induced scattering rates in a clear and transparent way.

Thus, we postulate that the dynamics of the QD-driven system can now be approximately described through

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i}{\hbar} \left( [H_S^{ef}, \rho(t)] + D_{ph}(\rho) \right) + \mathcal{L}(\rho) + \mathcal{L}_{ph}(\rho), \quad (34)$$

where  $L_{ph}(\rho)$  (ph refers to phonon) is given by

$$\begin{aligned} \mathcal{L}_{ph}(\rho) = & \mathcal{L}_{ph}^{Intp} + \frac{\Gamma_W^{\sigma_{11}}}{2} L(\sigma_{11}) + \frac{\Gamma_{ph}^{\sigma^+ a}}{2} L(\sigma^+ a) \\ & + \frac{\Gamma_{ph}^{\sigma^-}}{2} L(\sigma^-) + \frac{\Gamma_{ph}^{a^\dagger \sigma^-}}{2} L(a^\dagger \sigma^-) + \frac{\Gamma_{ph}^{\sigma^+}}{2} L(\sigma^+), \end{aligned} \quad (35)$$

with the super operator  $L(D)[\rho]$  is defined as  $L(D) = 2D\rho D^\dagger - D^\dagger D\rho - \rho D^\dagger D$ . On the other hand the term  $\mathcal{L}_{ph}^{Intp}$  describes the incoherent interpolation processes between weak coupling and polaronic theory and is given by

$$\begin{aligned} \mathcal{L}_{ph}^{Intp} = & \frac{\Gamma_{zy}^{\sigma_{11} \sigma^+}}{2} L_{ph}^{Intp}(\sigma_{11}, \sigma^+) + \hbar \Gamma_{zy}^{\sigma_{11} \sigma^-} L_{ph}^{Intp}(\sigma_{11}, \sigma^-) \\ & + \frac{\Gamma_{zy}^{\sigma_{11} \sigma^+ a}}{2} L_{ph}^{Intp}(\sigma_{11}, \sigma^+ a) + \frac{\Gamma_{zy}^{\sigma_{11} \sigma^- a^\dagger}}{2} L_{ph}^{Intp}(\sigma_{11}, \sigma^- a^\dagger) \\ & + \frac{\Gamma_{yz}^{\sigma^+ \sigma_{11}}}{2} L_{ph}^{Intp}(\sigma^+, \sigma_{11}) + \frac{\Gamma_{yz}^{\sigma^- \sigma_{11}}}{2} L_{ph}^{Intp}(\sigma^-, \sigma_{11}) \\ & + \frac{\Gamma_{yz}^{\sigma^+ a \sigma_{11}}}{2} L_{ph}^{Intp}(\sigma^+ a, \sigma_{11}) + \frac{\Gamma_{yz}^{\sigma^- a^\dagger \sigma_{11}}}{2} L_{ph}^{Intp}(\sigma^- a^\dagger, \sigma_{11}), \end{aligned} \quad (36)$$

The phonon mediated rates, which drive the effective Lindblad dynamics, are derived to be

$$\Gamma_W^{\sigma_{11}} = 2Re \left[ \int_0^\infty d\tau e^{2i\eta_x \tau} C_{zz}(\tau) \right] \quad (37)$$

$$\Gamma_{ph}^{\sigma^+ a / a^\dagger \sigma^-} = 2\langle B \rangle^2 g^2 Re \left[ \int_0^\infty d\tau e^{\pm \Delta_{cx} \tau} (e^{\phi(\tau)} - 1) \right], \quad (38)$$

$$\Gamma_{ph}^{\sigma^+ / \sigma^-} = 2\langle B \rangle^2 \eta_x^2 Re \left[ \int_0^\infty d\tau e^{\mp \Delta_{xL} \tau} (e^{\phi(\tau)} - 1) \right], \quad (39)$$

$$\Gamma_{zy}^{\sigma_{11} \sigma^\pm} = \mp 2\eta_x Im \left[ \int_0^\infty d\tau C_{zy} e^{\mp \Delta_{xL} \tau} \right], \quad (40)$$

$$\Gamma_{zy}^{\sigma_{11} \sigma^+ a / \sigma^- a^\dagger} = \mp 2g Im \left[ \int_0^\infty d\tau C_{zy} e^{\pm \Delta_{cx} \tau} \right], \quad (41)$$

$$\Gamma_{yz}^{\sigma^\pm \sigma_{11}} = \mp 2\eta_x Im \left[ \int_0^\infty d\tau C_{zy} e^{2i\eta_x \tau} \right], \quad (42)$$

$$\Gamma_{yz}^{\sigma^+ a / \sigma^- a^\dagger \sigma_{11}} = \mp 2g Im \left[ \int_0^\infty d\tau C_{zy} e^{2i\eta_x \tau} \right], \quad (43)$$

In addition to the phonon-induced Lindblad decay rates above, one also has phonon-mediated frequency shifts beyond the polaron shift. The effective Hamiltonian, describing the coherent part of the system evolution  $H_S^{ef}$  becomes

$$\begin{aligned} H_S^{ef} = & \hbar \Delta_{xL} \sigma^+ \sigma^- + \hbar \Delta_{cL} a^\dagger a + \langle B \rangle \zeta_x + \hbar \Delta_W^{\sigma_{11}} \sigma^+ \sigma^- \\ & + \Delta_{ph}^{\sigma^+ a} a^\dagger \sigma^- \sigma^+ a + \hbar \Delta_{ph}^{\sigma^-} \sigma^- \sigma^+ \\ & + \hbar \Delta_{ph}^{a^\dagger \sigma^-} a \sigma^+ \sigma^- a^\dagger + \hbar \Delta_{ph}^{\sigma^+} \sigma^+ \sigma^-. \end{aligned} \quad (44)$$

and the coherent product of interpolation between weak coupling and polaronic theory are given by

$$\begin{aligned} D_{ph} = & \hbar \Delta_{zy}^{\sigma_{11} \sigma^+} \mathfrak{D}_{ph}^{Intp}(\sigma_{11}, \sigma^+) + \hbar \Delta_{zy}^{\sigma_{11} \sigma^-} \mathfrak{D}_{ph}^{Intp}(\sigma_{11}, \sigma^-) \\ & + \hbar \Delta_{zy}^{\sigma_{11} \sigma^+ a} \mathfrak{D}_{ph}^{Intp}(\sigma_{11}, \sigma^+ a) + \hbar \Delta_{zy}^{\sigma_{11} \sigma^- a^\dagger} \mathfrak{D}_{ph}^{Intp}(\sigma_{11}, \sigma^- a^\dagger) \\ & + \hbar \Delta_{yz}^{\sigma^+ \sigma_{11}} \mathfrak{D}_{ph}^{Intp}(\sigma^+, \sigma_{11}) + \hbar \Delta_{yz}^{\sigma^- \sigma_{11}} \mathfrak{D}_{ph}^{Intp}(\sigma^-, \sigma_{11}) \\ & + \hbar \Delta_{yz}^{\sigma^+ a \sigma_{11}} \mathfrak{D}_{ph}^{Intp}(\sigma^+ a, \sigma_{11}) + \hbar \Delta_{yz}^{\sigma^- a^\dagger \sigma_{11}} \mathfrak{D}_{ph}^{Intp}(\sigma^- a^\dagger, \sigma_{11}), \end{aligned} \quad (45)$$

with

$$\Delta_W^{\sigma_{11}} = Im \left[ \int_0^\infty d\tau e^{2i\eta_x \tau} C_{zz}(\tau) \right] \quad (46)$$

$$\Delta_{ph}^{\sigma^+ a / a^\dagger \sigma^-} = \langle B \rangle^2 g^2 Im \left[ \int_0^\infty d\tau e^{\pm \Delta_{cx} \tau} (e^{\phi(\tau)} - 1) \right], \quad (47)$$

$$\Delta_{ph}^{\sigma^+ / \sigma^-} = \langle B \rangle^2 \eta_x^2 Im \left[ \int_0^\infty d\tau e^{\mp \Delta_{xL} \tau} (e^{\phi(\tau)} - 1) \right], \quad (48)$$

$$\Delta_{zy}^{\sigma_{11} \sigma^\pm} = \pm \eta_x Re \left[ \int_0^\infty d\tau C_{zy} e^{\mp \Delta_{xL} \tau} \right], \quad (49)$$

$$\Delta_{zy}^{\sigma_{11} \sigma^+ a / \sigma^- a^\dagger} = \pm g Re \left[ \int_0^\infty d\tau C_{zy} e^{\pm \Delta_{cx} \tau} \right], \quad (50)$$

$$\Delta_{yz}^{\sigma^\pm \sigma_{11}} = \pm \eta_x Re \left[ \int_0^\infty d\tau C_{zy} e^{2i\eta_x \tau} \right], \quad (51)$$

$$\Delta_{yz}^{\sigma^+ a / \sigma^- a^\dagger \sigma_{11}} = \pm g Re \left[ \int_0^\infty d\tau C_{zy} e^{2i\eta_x \tau} \right]. \quad (52)$$

## NUMERICAL RESULTS

To simulate the fluorescence spectrum of a QD coupled to a cavity and excited by a continuous wave (CW) laser resonance, is used the expression,

$$\begin{aligned} S_c(\omega) \propto & \lim_{t \rightarrow \infty} Re \left[ \int_0^\infty d\tau \left[ \langle a(t+\tau) a^\dagger(t) \rangle \right. \right. \\ & \left. \left. - \langle a(t+\tau) \rangle \langle a^\dagger(t) \rangle \right] e^{i(\omega_L - \omega)\tau} \right], \end{aligned} \quad (53)$$

where the correlation functions are obtained by the quantum regression formula [7]. The master equations ob-

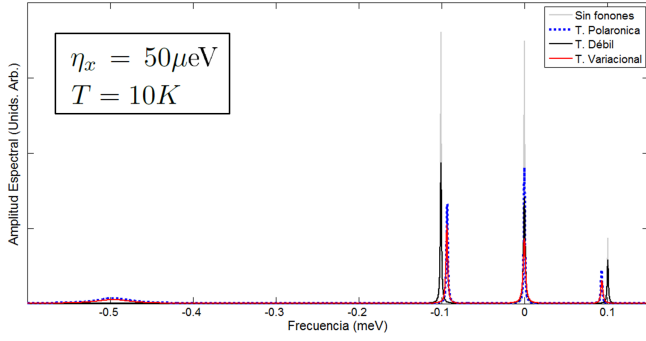


FIG. 2. Fluorescence spectra calculated using weak coupling theory (red curve), polaronic theory (blue dotted curve), variational master equation (black curve) and absence of phonon (gray curve). The pumping rate is in resonance with the exciton and is  $\eta_x = 50\mu\text{eV}$  while the system temperature is  $T = 10\text{K}$

tained within the different levels of approximation studied are solved numerically using the quantum optic *toolbox* for *MATLAB* developed by Tan [8], Assuming the stable pumping rate (i.e.  $\eta_x$  independent of time), the exciton initially in the base state and a variety of excitation equal to one [9].

To make our results comparable with the recent Mollow triplet experiments on semiconductor micropylars [2], we consider a detuning mode cavity  $\omega_c - \omega_x = -0.5\text{meV}$ , a radioactive decay rate  $\gamma = 2\mu\text{eV}$ , a pure-dephasing rate  $\gamma' = 1\mu\text{eV}$  and an emission rate of the cavity  $\kappa = 50\mu\text{eV}$ ; these values are similar to those given in the experiments [3]. As for phonon parameters, the typical parameters for InAs / GaAs QDs are used, with a cutoff frequency  $\omega_b = 2.2\text{ps}^{-1}$  y  $\alpha_p = 0.027\text{ps}^2$  [10, 11]. In all charts of this chapter, the frequency is taken with respect to the QD emission.

In this way we compare the weak coupling model, the polaronic theory and the variational theory are compared, together with the case in which the phonons are not included. In the figure 2 the system is subject to low rates of pumping and temperature. Under these conditions the three dissipative models differ little, predicting Mollow's triplet peaks noticeably less intense than the non-phonon case. By increasing the temperature of the system [figure 3], we see how the weak coupling model differs greatly from the polar and variational theories, overestimating the phonon bath induced decoherence processes. On the other hand, it is appreciable the renormalization of the pumping rate present in the polaronic and variational model. Thus, for a system whose temperature is high and with a moderate pumping rate, the polar and variational theories predict a similar dynamic

Finally, by raising the pumping rate by keeping the temperature low [figure 4], all three models differ markedly. Under these conditions the variational the-

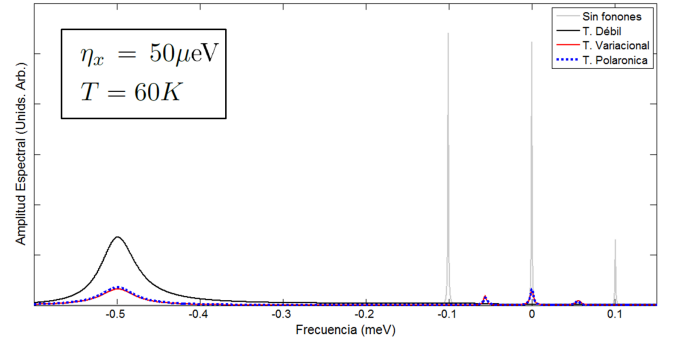


FIG. 3. Fluorescence spectra calculated using weak coupling theory (red curve), polaronic theory (blue dotted curve), variational master equation (black curve) and absence of phonon (gray curve). The pumping rate is in resonance with the exciton and is  $\eta_x = 50\mu\text{eV}$  while the system temperature is  $T = 60\text{K}$

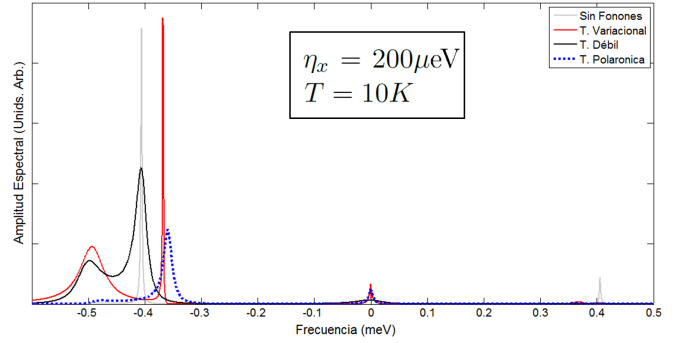


FIG. 4. Fluorescence spectra calculated using weak coupling theory (red curve), polaronic theory (blue dotted curve), variational master equation (black curve) and absence of phonon (gray curve). The pumping rate is in resonance with the exciton and is  $\eta_x = 200\mu\text{eV}$  while the system temperature is  $T = 10\text{K}$

ory interpolates the weak and polaronic coupling model, describing the relationship between the peak associated to the cavity mode and the left side peak of the Mollow triplet similar to that predicted by the weak model, while the peak Central and left side are described in a manner analogous to that posed by the polaronic theory. In addition it can be seen how the renormalization of the pumping rate of the polaronic and variational model differ significantly, since in the latter, besides being susceptible to temperature, it is dependent on the variational factor " $F(\omega)$ " [see equation (19)].

## SUMMARY

Thus, we derive an optimized master equation, based on the application of the polaronic form transformation, but with the mode-dependent displacements of the phonon determined by a variational process. In this way

a theory was obtained that was flexible enough to encompass weak and polaronic coupling methods within the limits appropriate to each one. Thus, the variational master equation proved to be appropriate in a much wider range of temperature, pumping rate and QD-cavity coupling.

## ACKNOWLEDGEMENTS

The authors acknowledge the Research Division of Universidad Pedagógica y Tecnológica de Colombia for financial support.

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